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SPATIAL GEODETIC TRIANGULATION

by

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FOREIGN TECHNOLOGY DIVISION

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SPATIAL
GEODETIC TRIANGULATION

Georgi Zlatanov

Introduction

In recent years a rapid development of science and technology is observable. In the field of geodesy this development is characterized by:

- The introduction and rapid spread of electromagnetic stadia
- The individuation of a new geodetic field - cosmic geodesy
- The versatile implementation of electronic technology to geodetic techniques

These new elements in geodesic theory and practice have enabled new methods for the reciprocal establishing of the location of points on the surface of the Earth. These methods are essentially different from classical methods.

In classical geodesy all dimensions were divided into two basic groups: horizontal dimensions and vertical dimensions. Classical geodesy required a reference surface, upon which were reduced all measurement results.

The problem of measurement reduction was one of the basic problems of classical geodesy. The accuracy with which reduction corrections could be determined constituted, in many cases, a barrier determining the accuracy of the geodesic net.

Modern technology allowed that barrier to be overcome. An important role in this was played by satellite geodesy,

which in a short time achieved a high level of development. An essential share in this development was contributed by Polish geodesists.

As is well known, cosmic triangulation can be thickened,[1] with the aid of the so-called balloon triangulation. As a result of this thickening the geocentric coordinates of a large number of points on the Earth's surface can be gained. Consequently, it is entirely possible to obtain the geocentric coordinates of points lying 200-300 km apart, without the necessity of reduction (in the classical geodesic sense) results of direct measurement, and this without using a reference surface. These new possibilities, essentially differing from classical means, require the discovery of new methods for establishing basic geodesic nets.

In this current work a method for the mathematical working out of the geodetic net is considered, based on three-dimensional geodesy. Adjustment and calculation of the net is performed on a three-dimensional Cartesian coordinate system, and not on a reference surface. By this method the difficulties resulting from the reduction problem are avoided. There arises, however, the necessity of utilizing physical reduction, and the especial need to consider the effect of refraction.

Due to the character of the work, it is assumed, that the angles of refraction are determined beforehand (e.g., with the aid of a refractometer), and thus, that the zenith distances are free from the influence of refraction.

In three-dimensional geodesy it's necessary to make an aggregate adjustment for the effects of the measurement of the vertical and horizontal angles. There thus arises the possibility of making calculations in a three-dimensional, Cartesian coordinate system. No difficulties are presented, in this case, including the effects of length measurements.

The idea of three-dimensional non-reduced geodesy arose in the middle of the last century in the work of Willarseau and Bruns. At that time, however, its development could not be continued. Only in the 1950's do we encounter the first systematic investigations dedicated to three-dimensional geodetic. The groundwork for a geometric approach to the question of determining the figure of the Earth was laid by M. S. Mołodieński. A series of basic mathematical equations for three-dimensional geodesy can be found in the work of Hotin and Dufor.

After formulating the basic laws of cosmic geodesy, and after working out the geometric methods for investigating the Earth's figure, there began researches regarding problems arising in conducting local spatial triangulation.

The set of problems included here can be divided into two groups:

1. The effects of refraction on the results of zenith distance measurements
2. Methods of adjustment for spatial networks

Whereas the question of the effect of refraction finds a diverse and occasionally contradictory variety of interpretations, methods for adjustment of spatial networks can be assigned to two groups:

1. Separate adjustment of the vertical and horizontal measurement results

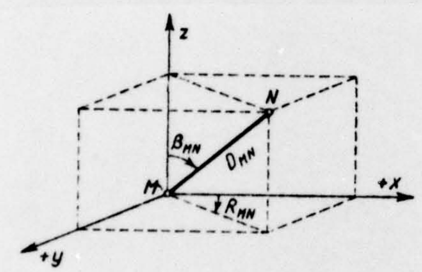
2. Aggregate, i.e., three-dimensional adjustment of vertical and horizontal measurement results.

By separately adjusting these first is determined the horizontal coordinates of the network points, and next the third coordinate, together with the unknown perpendicular deviation. The means of adjustment, belonging to this group, do not differ essentially from the classical geodetic methods discussed in textbooks. They represent a single minor modification of the known methods. The essential difference in relation to classical means, is shown by the methods for establishing aggregate adjustment of the horizontal and vertical angle measurement results. In this adjustment perpendicular deviations also play a role. It results that measurements carried out on the surface of the Earth are directly connected with the direction of the perpendicular line at the observation stations. There arises thus the necessity to reduce them to a uniform system of coordinates.

In the current work two algorithms for spatial triangulation adjustment are presented; in the first of these multigroup adjustments by the conditional method is used with unknowns, and in the second -- the parameter method of adjustment for correlated quantities.

I

Whereas in cosmic geodesy the axes of coordinates of a topocentric system of coordinates is parallel to the axis of a geocentric system, in the case under examination -- topocentric systems of coordinates are used. Closely connected with the positions of the theodolite at the time of observation. Thus, for example, the topocentric coordinate system $(\overset{x}{\cancel{X}}, \overset{y}{\cancel{Y}}, \overset{z}{\cancel{Z}})_M$ (drawing 1) can be defined as follows: axis Z is directed towards the zenith (along the horizontal axis of the instrument); axis X lies in a horizontal plane and is directed towards a zero indicator of the horizontal circle; and axis $\overset{y}{\cancel{Y}}$ is turned 90° in relation to axis $\overset{x}{\cancel{X}}$, in a clockwise direction. Such a topocentric coordinate system is a left-handed system.



R_{MN} - horizontal angle between axis $\cancel{X}$ x and direction M-N

β_{MN} - zenith distance of direction M-N

D_{MN} - distance M-N

The position of the arbitrary point N in the topocentric coordinate system for station M can be defined by polar coordinates (R, β, D) and by the rectangular coordinates $(\overset{x}{\cancel{X}}, \overset{y}{\cancel{Y}}, \overset{z}{\cancel{Z}})_M$.

The relationship between these coordinates is as follows:

$$\begin{aligned} x_N^{(M)} &= D_{MN} \cos R_{MN} \sin\beta_{MN}; \\ y_N^{(M)} &= D_{MN} \sin R_{MN} \sin\beta_{MN}; \\ z_N^{(M)} &= D_{MN} \cos\beta_{MN}. \end{aligned} \qquad (1.1)$$

From (1.1) we can obtain the reciprocal relationship:

$$D_{MN} = +\sqrt{x_N^{(M)2}+y_N^{(M)2}+z_N^{(M)2}};$$
$$\operatorname{tg} R_{MN} = \frac{y_N^{(M)}}{x_N^{(M)}};$$
$$\cos\beta_{MN} = \frac{z_N^{(M)}}{D_{MN}} = \frac{z_N^{(M)}}{+\sqrt{x_N^{(M)2}+y_N^{(M)2}+z_N^{(M)2}}}. \qquad (1.2)$$

For the direction cosines MN in a local coordinate system we have:

$$a_{MN} = \frac{x_N^{(M)}}{D_{MN}} = \cos R_{MN}\cdot\sin\beta_{MN};$$
$$b_{MN} = \frac{y_N^{(M)}}{D_{MN}} = \sin R_{MN}\cdot\sin\beta_{MN};$$
$$c_{MN} = \frac{z_N^{(M)}}{D_{MN}} = \cos\beta_{MN}. \qquad (1.3)$$

If we designate the versors of the topocentric system of coordinates by $(\vec{e}_1, \vec{e}_2, \vec{e}_3)_M$, then for the unit direction vector MN, after using the general cracovian calculus operators, we can write:

$$\vec{r}_{MN} = a_{MN}\vec{e}_1^{(M)}+b_{MN}\vec{e}_2^{(M)}+c_{MN}\vec{e}_3^{(M)} = \mathbf{a}_{MN}\cdot\tau\mathbf{e}_M, \qquad (1.4)$$

Vector $\vec{r}_{MN}$ can also be presented in a geocentric coordinate system (X, Y, Z). If the versors of this system are designated by $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$, then $\vec{r}_{MN}$ can be written as follows:

$$\vec{r}_{MN} = A_{MN}\cdot\vec{E}_1+B_{MN}\cdot\vec{E}_2+C_{MN}\cdot\vec{E}_3 = \mathbf{A}_{MN}\cdot\tau\mathbf{E}, \qquad (1.5)$$

where:

$$A_{MN} = \frac{X_N-X_M}{D_{MN}},$$
$$B_{MN} = \frac{Y_N-Y_M}{D_{MN}},$$
$$C_{MN} = \frac{Z_N-Z_M}{D_{MN}}$$
$$D_{MN} = +\sqrt{(X_N-X_M)^2+(Y_N-Y_M)^2+(Z_N-Z_M)^2}. \qquad (1.6)$$

From (1.4) and (1.5) it results:

$$\bar{r}_{MN} = a_{MN} \cdot \tau e_M = A_{MN} \cdot \tau E. \qquad (1.7)$$

We accept that the relation between e_M and E is represented in the following form:

$$E = u_M \cdot \tau e_M, \qquad (1.8)$$
(3.3)

where U_M is an orthogonal cracovian. It is designated by three topocentric orientation angles of the topocentric coordinate system, namely:

- $\alpha_0(M)$ - the astronomical azimat of axis X_M (of a direction corresponding to the zero indicator on the horizontal circle of the theodolite at point M),
- $\varphi(M)$ - astronomical width at point M,
- $\lambda(M)$ - astronomical length at point M,

For elements U_M the following relations result:

$$\begin{aligned}
u_{11} &= -\cos\alpha_0 \sin\varphi \cos\lambda - \sin\alpha_0 \sin\lambda,\\
u_{21} &= -\cos\alpha_0 \sin\varphi \sin\lambda + \sin\alpha_0 \cos\lambda,\\
u_{31} &= \cos\alpha_0 \cos\varphi;\\
u_{12} &= \sin\alpha_0 \sin\varphi \cos\lambda - \cos\alpha_0 \sin\lambda,\\
u_{22} &= \sin\alpha_0 \sin\varphi \sin\lambda + \cos\alpha_0 \cos\lambda,\\
u_{32} &= -\sin\alpha_0 \cos\varphi;\\
u_{13} &= \cos\varphi \cos\lambda,\\
u_{23} &= \cos\varphi \sin\lambda,\\
u_{33} &= \sin\varphi.
\end{aligned} \qquad (1.9)$$

In these formulae the factor M is disregarded.

From (1.7) and (1.8) it results:

$$\underset{(1.3)}{a_{MN}} = \underset{(1.3)}{A_{MN}} \cdot \underset{(3.3)}{u_M}, \qquad (1.10)$$

$$\underset{(1.3)}{A_{MN}} = \underset{(1.3)}{a_{MN}} \cdot \underset{(3.3)}{u_M} \qquad (1.10a)$$

The form of the developed formula (1.10) appears as follows:

$$\begin{aligned} a_{MN} &= A_{MN}u_{11}^{(M)} + B_{MN}u_{21}^{(M)} + C_{MN}u_{31}^{(M)}, \\ b_{MN} &= A_{MN}u_{12}^{(M)} + B_{MN}u_{22}^{(M)} + C_{MN}u_{32}^{(M)}, \\ c_{MN} &= A_{MN}u_{13}^{(M)} + B_{MN}u_{23}^{(M)} + C_{MN}u_{33}^{(M)}. \end{aligned} \qquad (1.11)$$

The relationship (1.11) presents these conditional equations between the results of observations R_{MN}, β_{MN}, and coordinates $(X, Y, Z)_{MN}$, as well as the orientation angles $(\alpha_0, \varphi, \lambda)_M$. Between these three equations there ~~arises~~ occurs the ~~relationship~~ dependence ~~(equation)~~; $a^2_{MN}+b^2_{MN}+c^2_{MN}=1$, ~~or we can include~~ only two of these equations, for example, the terms a_{MN} and C_{MN}, can be incorporated in the adjustment.

Before approaching adjustment, we must present relationship (1.11) in linear form.

Let us use these notations:

$$\left.\begin{aligned} &R_{MN} = R'_{MN} + V_{R_{MN}}; \quad \beta_{MN} = \beta'_{MN} + V_{\beta_{MN}}; \\ &a'_{MN} = \cos R'_{MN} \sin \beta'_{MN}, \\ &b'_{MN} = \sin R'_{MN} \sin \beta'_{MN}, \\ &c'_{MN} = \cos \beta'_{MN}; \end{aligned}\right\} \qquad (1.12a)$$

$$\left.\begin{aligned} &\alpha_{M_0} = \alpha^0_{M_0} + d\alpha_M, \quad \varphi_M = \varphi^0_M + d\varphi_M, \quad \lambda_M = \lambda^0_M + d\lambda_M; \\ &X_K = X^0_K + dX_K, \quad Y_K = Y^0_K + dY_K, \quad Z_K = Z^0_K + dZ_K \quad (K = M, N); \end{aligned}\right\} \qquad (1.12b)$$

$$\left.\begin{aligned} &A^0_{MN} = \frac{X^0_N - X^0_M}{D^0_{MN}}, \quad B^0_{MN} = \frac{Y^0_N - Y^0_M}{D^0_{MN}}, \quad C^0_{MN} = \frac{Z^0_N - Z^0_M}{D^0_{MN}}, \\ &u^{0(M)}_{ij} = u^{(M)}_{ij}(\alpha^0_{M_0}, \varphi^0_M, \lambda^0_M). \end{aligned}\right\} \qquad (1.12c)$$

In the above relationships the indicator "prim" designates measured values, and indicator "0" -- approximate values.

The linear form of equations (1.11) is as follows:

$$
\begin{aligned}
&\frac{\partial a'_{MN}}{\partial R'_{MN}} \cdot V_{R_{MN}} + \frac{\partial a'_{MN}}{\partial R'_{MN}} \cdot V_{\beta_{MN}} - \left(u^0_{11} \frac{\partial A_{MN}}{\partial X_M} + u^0_{21} \frac{\partial B_{MN}}{\partial X_M} + u^0_{31} \frac{\partial C_{MN}}{\partial X_M} \right) (dX_M - dX_N) + \\
&- \left(u^0_{11} \frac{\partial A_{MN}}{\partial Y_M} + u^0_{21} \frac{\partial B_{MN}}{\partial Y_M} + u^0_{31} \frac{\partial C_{MN}}{\partial Y_M} \right) (dY_M - dY_N) - \left(u^0_{11} \frac{\partial A_{MN}}{\partial Z_M} + u^0_{21} \frac{\partial B_{MN}}{\partial Z_M} + u^0_{31} \frac{\partial C_{MN}}{\partial Z_M} \right) (dZ_M - dZ_N) + \\
&- \left(A^0_{MN} \frac{\partial u_{11}}{\partial \alpha_0^{(M)}} + b^0_{MN} \frac{\partial u_{21}}{\partial \alpha_0^{(M)}} + C^0_{MN} \frac{\partial u_{31}}{\partial \alpha_0^{(M)}} \right) d\alpha_0^{(M)} - \left(A^0_{MN} \frac{\partial u_{11}}{\partial \varphi^{(M)}} + B^0_{MN} \frac{\partial u_{21}}{\partial \varphi^{(M)}} + C^0_{MN} \frac{\partial u_{31}}{\partial \varphi^{(M)}} \right) d\varphi_0^{(M)} + \\
&- \left(A^0_{MN} \frac{\partial u_{11}}{\partial \lambda^{(M)}} + B^0_{MN} \frac{\partial u_{21}}{\partial \lambda^{(M)}} + C^0_{MN} \frac{\partial u_{31}}{\partial \lambda^{(M)}} \right) d\lambda^{(M)} + W^1_{MN} = 0,
\end{aligned}
\tag{1.13a}
$$

$$
\begin{aligned}
&\frac{\partial b'_{MN}}{\partial R'_{MN}} \cdot V_{R_{MN}} + \frac{\partial b'_{MN}}{\partial \beta_{MN}} \cdot V_{\beta_{MN}} - \left(u^0_{12} \frac{\partial A_{MN}}{\partial X_M} + u^0_{22} \frac{\partial B_{MN}}{\partial X_M} + u^0_{32} \frac{\partial C_{MN}}{\partial X_M} \right) (dX_M - dX_N) + \\
&- \left(u^0_{12} \frac{\partial A_{MN}}{\partial Y_M} + u^0_{22} \frac{\partial B_{MN}}{\partial Y_M} + u^0_{32} \frac{\partial C_{MN}}{\partial Y_M} \right) (dY_M - dY_N) - \left(u^0_{12} \frac{\partial A_{MN}}{\partial Z_M} + u^0_{22} \frac{\partial B_{MN}}{\partial Z_M} + u^0_{32} \frac{\partial C_{MN}}{\partial Z_M} \right) (dZ_M - dZ_N) + \\
&- \left(A^0_{MN} \frac{\partial u_{12}}{\partial \alpha_0^{(M)}} + B^0_{MN} \frac{\partial u_{22}}{\partial \alpha_0^{(M)}} + C^0_{MN} \frac{\partial u_{32}}{\partial \alpha_0^{(M)}} \right) d\alpha_0^{(M)} - \left(A^0_{MN} \frac{\partial u_{12}}{\partial \varphi^{(M)}} + B^0_{MN} \frac{\partial u_{22}}{\partial \varphi^{(M)}} + C^0_{MN} \frac{\partial u_{32}}{\partial \varphi^{(M)}} \right) d\varphi^{(M)} + \\
&- \left(A^0_{MN} \frac{\partial u_{12}}{\partial \lambda^{(M)}} + B^0_{MN} \frac{\partial u_{22}}{\partial \lambda^{(M)}} + C^0_{MN} \frac{\partial u_{32}}{\partial \lambda^{(M)}} \right) d\lambda^{(M)} + W^2_{MN} = 0,
\end{aligned}
\tag{1.13b}
$$

$$
\begin{aligned}
&\frac{\partial c'_{MN}}{\partial \beta_{MN}} \cdot V_{\beta_{MN}} - \left(u^0_{13} \frac{\partial A_{MN}}{\partial X_M} + u^0_{23} \frac{\partial B_{MN}}{\partial X_M} + u^0_{33} \frac{\partial C_{MN}}{\partial X_M} \right) (dX_M - dX_N) + \\
&- \left(u^0_{13} \frac{\partial A_{MN}}{\partial Y_M} + u^0_{23} \frac{\partial B_{MN}}{\partial Y_M} + u^0_{33} \frac{\partial C_{MN}}{\partial Y_M} \right) (dY_M - dY_N) - \left(u^0_{13} \frac{\partial A_{MN}}{\partial Z_M} + u^0_{23} \frac{\partial B_{MN}}{\partial Z_M} + u^0_{33} \frac{\partial C_{MN}}{\partial Z_M} \right) (dZ_M - dZ_N) + \\
&- \left(A^0_{MN} \frac{\partial u_{13}}{\partial \alpha_0^{(M)}} + B^0_{MN} \frac{\partial u_{23}}{\partial \alpha_0^{(M)}} + C^0_{MN} \frac{\partial u_{33}}{\partial \alpha_0^{(M)}} \right) d\alpha_0^{(M)} - \left(A^0_{MN} \frac{\partial u_{13}}{\partial \varphi^{(M)}} + B^0_{MN} \frac{\partial u_{23}}{\partial \varphi^{(M)}} + C^0_{MN} \frac{\partial u_{33}}{\partial \varphi^{(M)}} \right) d\varphi^{(M)} + \\
&- \left(A^0_{MN} \frac{\partial u_{13}}{\partial \lambda^{(M)}} + B^0_{MN} \frac{\partial u_{13}}{\partial \lambda^{(M)}} + C^0_{MN} \frac{\partial u_{33}}{\partial \lambda^{(M)}} \right) d\lambda^{(M)} + W^3_{MN} = 0.
\end{aligned}
\tag{1.13c}
$$

The partial derivatives occuring in equation (1.13) are expressed as follows:

$$\frac{\partial a'_{MN}}{\partial R'_{MN}} = -\sin R'_{MN}\sin\beta'_{MN}, \qquad \frac{\partial a'_{MN}}{\partial \beta'_{MN}} = \cos R'_{MN}\cos\beta'_{MN};$$

$$\frac{\partial b'_{MN}}{\partial R'_{MN}} = \cos R'_{MN}\sin\beta'_{MN}, \qquad \frac{\partial b'_{MN}}{\partial \beta_{MN}} = \sin R'_{MN}\cos\beta'_{MN}; \tag{1.14a}$$

$$\frac{\partial C'_{MN}}{\partial \beta'_{MN}} = -\sin\beta'_{MN}.$$

$$\frac{\partial A_{MN}}{\partial X_M} = -\frac{\Delta Y^{02}_{MN}+\Delta Z^{02}_{MN}}{D^{03}_{MN}}; \qquad \frac{\partial B_{MN}}{\partial X_M} = \frac{\Delta Y^0_{MN}\cdot\Delta X^0_{MN}}{D^{03}_{MN}}; \qquad \frac{\partial C_{MN}}{\partial X_M} = \frac{\Delta Z^0_{MN}\cdot\Delta X^0_{MN}}{D^{03}_{MN}};$$

$$\frac{\partial A_{MN}}{\partial Y_M} = \frac{\Delta Y^0_{MN}\cdot\Delta X^0_{MN}}{D^{03}_{MN}}; \qquad \frac{\partial B_{MN}}{\partial Y_M} = -\frac{\Delta X^{02}_{MN}+\Delta Z^{02}_{MN}}{D^{03}_{MN}}; \qquad \frac{\partial C_{MN}}{\partial Y_M} = \frac{\Delta Z^0_{MN}\cdot\Delta Y^0_{MN}}{D^{03}_{MN}}; \tag{1.14b}$$

$$\frac{\partial A_{MN}}{\partial Z_M} = \frac{\Delta X^0_{MN}\cdot\Delta Z^0_{MN}}{D^{03}_{MN}}; \qquad \frac{\partial B_{MN}}{\partial Z_M} = \frac{\Delta Y^0_{MN}\cdot\Delta Z^0_{MN}}{D^{03}_{MN}}; \qquad \frac{\partial C_{MN}}{\partial Z_M} = -\frac{\Delta X^{02}_{MN}+\Delta Y^{02}_{MN}}{D^{03}_{MN}};$$

$$\frac{\partial u_{11}}{\partial \alpha_0} = +u^0_{21}, \qquad \frac{\partial u_{21}}{\partial \alpha_0} = -u^0_{12}, \qquad \frac{\partial u_{31}}{\partial \alpha_0} = 0;$$

$$\frac{\partial u_{12}}{\partial \alpha_0} = +u^0_{22}, \qquad \frac{\partial u_{22}}{\partial \alpha_0} = -u_{11}, \qquad \frac{\partial u_{32}}{\partial \alpha_0} = 0; \tag{1.14c}$$

$$\frac{\partial u_{13}}{\partial \alpha_0} = +u^0_{23}, \qquad \frac{\partial u_{23}}{\partial \alpha_0} = -u_{13}, \qquad \frac{\partial u_{33}}{\partial \alpha_0} = 0;$$

$$\frac{\partial u_{11}}{\partial \varphi} = -u^0_{13}\cos\lambda^0, \qquad \frac{\partial u_{21}}{\partial \varphi} = -u^0_{23}\cos\lambda^0, \qquad \frac{\partial u_{31}}{\partial \varphi} = -u^0_{33}\cos\lambda^0;$$

$$\frac{\partial u_{12}}{\partial \varphi} = -u^0_{13}\sin\lambda^0, \qquad \frac{\partial u_{22}}{\partial \varphi} = -u^0_{23}\sin\lambda^0, \qquad \frac{\partial u_{32}}{\partial \varphi} = -u^0_{33}\sin\lambda^0; \tag{1.14d}$$

$$\frac{\partial u_{13}}{\partial \varphi} = -u^0_{33}\cos\alpha_0, \qquad \frac{\partial u_{23}}{\partial \varphi} = u^0_{33}\sin\alpha_0, \qquad \frac{\partial u_{33}}{\partial \varphi} = \cos\varphi^0.$$

$$\frac{\partial u_{11}}{\partial \lambda} = -u^0_{12}, \qquad \frac{\partial u_{21}}{\partial \lambda} = -u^0_{22}, \qquad \frac{\partial u_{31}}{\partial \lambda} = -u^0_{32};$$

$$\frac{\partial u_{12}}{\partial \lambda} = +u^0_{11}, \qquad \frac{\partial u_{22}}{\partial \lambda} = +u^0_{21}, \qquad \frac{\partial u_{32}}{\partial \lambda} = u^0_{31}; \tag{1.14e}$$

$$\frac{\partial u_{13}}{\partial \lambda} = 0, \qquad \frac{\partial u_{23}}{\partial \lambda} = 0, \qquad \frac{\partial u_{33}}{\partial \lambda} = 0.$$

$$W^1_{MN} = a'_{MN} - A^0_{MN}u^0_{11} - B^0_{MN}u^0_{21} - C^0_{MN}u^0_{31},$$

$$W^2_{MN} = b'_{MN} - A^0_{MN}u^0_{12} - B^0_{MN}u^0_{22} - C^0_{MN}u^0_{32}, \tag{1.14f}$$

$$W^3_{MN} = c'_{MN} - A^0_{MN}u^0_{13} - B^0_{MN}u^0_{23} - C^0_{MN}u^0_{33}.$$

Utilizing the aid of EMC, the numerical values of these derivatives can be easily determined by the numerical differentiation method. In the case of the zenith distance not being freed from the effect of refraction beforehand, it is necessary to include in the adjustment the coefficient of refraction, K_M^k, as an unknown. To this end in equation (1.13) it is necessary to use a substitution:

$$V_{\beta_{MN}} = V'_{\beta_{MN}} + \frac{1}{2}\frac{D_{MN}}{R_{\acute{s}r}} \cdot k_M, \tag{1.15}$$

Where: $V'_{\beta_{MN}}$ - correction for measured value β'_{MN}
K_M - refraction coefficient for position M
$R_{\acute{s}r}$ - mean radius of the Earth

As we already know, for each direction MN it is necessary to employ equations, using two among the three relationships (1.13). Using the conditional method for multi-group adjustment with unknowns (8), this pair of equations will be treated as a separate group.

From each pair of conditional equations two are designated for equivalent corrections, relating to [WORD UNKNOWN] and independent fictitious measurements. These equations then undergo a general operation, as a result of which all unknowns $(X, Y, Z, \varphi, \lambda, \alpha_0)_M$ are obtained, also the corrections V_R, V_β for each measured direction.

We consider that including in the adjustment $\alpha_0, \varphi, \lambda$ for each position, increases to a large degree the dimensions of the system of normal equations, and therefore it would be worthwhile to eliminate these unknowns beforehand.

II

A problem for geodetic nets installed for use in hydraulic construction in mountainous regions, or in the construction of long tunnels is assuring high accuracy in the correlated position of a network of points.

Using the classical methods of determining the effects of the measurement of angles and lengths it is essential to know exactly the deviations from the perpendicular and the geoid interstices from the elipsoid.

Introduction to geodetic calculations of modern concepts allows the analytic presentations of the nets elements in three-dimensional Cartesian space. Thanks to this, the necessity of including corrections for measured quantities in order to transfer to a reference surface, is eliminated.

For the purpose of eliminating the effect of errors in the initial data, nets of high precision, the rules are regarded as independent, i.e., not connected with given points.

The adjustment algorithm considered in Part I requires the existence of at least three connected points, and thus can not be used for adjustment of an independent net.

Another algorithm is presented below, which can be used equally in the consolidation of spatial nets, and in the adjustment of independent nets.

In order to adjust local spatial nets, a basic problem is designating suitable initial data, which would determine the orientation elements of the net without its distortion. There are six orientation elements. They are the coordinates of the basic point $(\overset{z,\, y,\, x}{\cancel{X, Y, X}})_0$ and the three Euler angles. To these six orientation elements we must add the net scale.

The above seven parameters can be replaced by seven other independent quantities. There can be, for example, the rectangular coordinates of two points and one coordinate of the third point. With the existence of three data points, two supernumery elements then appear, which cause a net deformation.

The designation of only one geocentric coordinate for the net point in order to eliminate its deformation, presents great difficulties. The conclusion results, that the means of adjustment, well known in the literature, which uses geocentric coordinates (X, Y, Z) cannot be used in relation to unconnected spatial nets.

For adjusting local spatial nets the conditional method can be used, but great difficulty arises in this regard in setting-up conditional equations. In regard to this, in this work we have concentrated on the parameter method. The unknowns (X, Y, Z) are converted in advance to the geodetic coordinates (B, L, H).

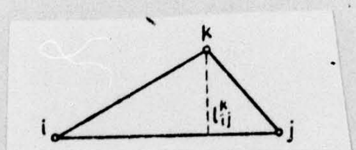
Adjusting the net begins by setting up the equations expressing certain quantities, independent of the orientation station. The first time -- as a function of geocentric coordinate stations and observed net point, and the second time -- as a function of horizontal and vertical angles, measured at the same station. The functions spoken of should be linearly independent and should simultaneously define the position of the directions measured at a given station.

In order to make the selection of suitable functions, let us consider the directions from station M to observed points I, J, and K.

Let us imagine, that point M is the center of a sphere of elementary radius. Points i, j, and k (Drawing 2) are then points intersecting directions MI, MJ, and MK from the surface of the sphere. The straight sections in Drawing 2 are in reality arcs of large circles, corresponding to spatial angles. Bearing in mind that points M, I, J, and K are points found on the Earth's surface, the spherical triangle (i, j, k) is very oblate (the degree of angle at point K is nearly 180°). In this situation the spherical distances (i,j), (i,k), and (k,j) are almost linearly dependent, or

$$(i,j) \approx (i,k)+(k,j), \tag{2.1}$$

and thus they can not reliably determine the position of directions MI, MJ, and MK.



Drawing 2

It also results that from three noted spherical distances for adjustment, only two are used, for example, (i,k) and (k,j). These quantities complement the spherical altitude L^{ki}_{ji} leading from point R.

It also results that from the three noted spherical distances for adjustment, only two are used, for example, (i,k) and (k,j). These quantities complement the spherical altitude $L^{ki}_{\cancel{ji}\; ij}$ leading from point R.

Let us designate

$$
\begin{aligned}
(s_{ik}) &= \cos(i,k), \\
(s_{kj}) &= \cos(k,j), \\
(h^k_{ij}) &= \sin(l^k_{ij}).
\end{aligned}
\qquad (2.2)
$$

These quantities (2.2) can be expressed by the elementary vectors $\vec{r}_i$, $\vec{r}_j$, and $\vec{r}_k$, for directions MI, MJ, and MK.

Then we have

$$
\begin{aligned}
(s_{ik}) &= (\vec{r}_i, \vec{r}_k), \\
(s_{kj}) &= (\vec{r}_k, \vec{r}_j), \\
(h^k_{ij}) &= \frac{(\vec{r}_i, \vec{r}_j, \vec{r}_k)}{|(\vec{r}_i \times \vec{r}_j)|}
\end{aligned}
\qquad (2.3)
$$

For quantities (2.3) the following dependencies are appropriate:

$$
\begin{aligned}
(s_{ik}) &= (s_{ki}), \\
(s_{kj}) &= (s_{jk}), \\
(h^k_{ij}) &= -(h^k_{ij}).
\end{aligned}
\qquad (2.4)
$$

Using quantities of type (s) and (h) as adjustment quantities does not create any essential difficulties. The important concern here is the correct designation of the correlation cracovian of these quantities, and considering this cracovian during the adjustment itself. (6)

The number of quantities of type (s) and (h) for examining station M, in an example when the horizontal directions are measured, and the zenith angles to points ~~K~~ k will equal (2k - 3), in which (k - 1) -- of type (s) and (k - 2) -- of type (h).

In setting-up the correction equations for quantities of type (s) and (h) formulae are used which present elementary vectors $\vec{r}_i$, $\vec{r}_j$, and $\vec{r}_k$ in a local, (1.4) and geocentric (1.5) system.

Keeping the local system in mind, we have:

$$(s_{ik})_{lok} = F_s^l(\beta_i, \beta_k; R_i, R_k), \tag{2.5a}$$

$$(h_{ij}^k)_{lok} = F_h^l(\beta_i, \beta_j, \beta_k; R_i, R_j, R_k). \tag{2.5b}$$

and for the geocentric system:

$$(s_{ik})_{geo} = F_s^g(X_M, Y_M, Z_M; X_i, Y_i, Z_i; X_k, Y_k, Z_k) = \Phi_s^g(B_M, L_M, H_M; B_i, L_i, H_i; B_k, L_k, H_k); \tag{2.6a}$$

$$(h_{ij}^k)_{geo} = F_h^g(X_M, Y_M, Z_M; X_i, Y_i, Z_i; X_j, Y_j, Z_j; X_k, Y_k, Z_k) = \Phi_h^g(B_M, L_M, H_M; B_i, L_i, H_i; B_j, L_j, H_j; B_k, L_k, H_k). \tag{2.6b}$$

The geocentric coordinates (X, Y, Z)p and geodetic (B, L, H)p for P = I, J, K, M, are connected with the well-known formula:

$$\begin{aligned} X_P &= (N_P + H_P)\cos B_P \cos L_P, \\ Y_P &= (N_P + H_P)\cos B_P \sin L_P, \\ Z_P &= (N_P + H_P)\sin B_P - e^2 N_P \sin B_P, \end{aligned} \tag{2.7}$$

where $N_p = a(1 - e^2\sin^2 B_p)^{-1/2}$ is at the radius of curvature of a transverse section, e -- is the eccentric of the meredional elipse.

Functions (2.5) and (2.6) are obtained as a result of (2.3) and by considering (1.5) and (1.6). The final form of the functional dependencies Φ^g_s and Φ^g_h, in which appear thegeodetic coordinates (B, L, H), points (M, I, J, K) is determined resulting from (2.3), (1.5), and (1.6), and converting in (1.6) coordinates (X, Y, Z) to geodetic (B, L, H) with the aid of relations (2.7).

In order to obtain the correction equation, these substitutions are used:

$$\beta = \beta' + V_\beta, \qquad R = R' + V_R; \tag{2.8a}$$

$$X = X^0 + dX, \qquad Y = Y^0 + dY, \qquad Z = Z^0 + dZ; \tag{2.8b}$$

$$B = B^0 + dB, \qquad L = L^0 + dL, \qquad H = H^0 + dH; \tag{2.8c}$$

and (2.5a) and (2.6) develop in the Taylor series, limited to linear expressions.

Then we have

$$(s_{ik})_{\mathrm{tok}} = (s_{ik})' + V(s_{ik}), \tag{2.9a}$$

$$(h^k_{ij})_{\mathrm{tok}} = (h^k_{ij})' + V(h^k_{ij}), \tag{2.9b}$$

where

$$(s_{ik})' = F^1_s(\beta'_i, \beta'_k; R'_i, R_k), \tag{2.10a}$$

$$(h^k_{ij})' = F^1_h(\beta'_i, \beta'_j, \beta'_k; R'_i, R'_j, R'_k). \tag{2.10b}$$

$V(s_{ik})$ and $V(h^k_{ij})$ are prospective corrections for quantities (s_{ik}) and (j^k_{ij}) (functions of directly measured quantities, subject to adjustment), whose differential forms are as follows:

$$V(s_{ik}) = \frac{\partial(s_{ik})}{\partial R_i} V_{R_i} + \frac{\partial(s_{ik})}{\partial R_k} V_{R_k} + \frac{\partial(s_{ik})}{\partial \beta_i} V_{\beta_i} + \frac{\partial(s_{ik})}{\partial \beta_k} V_{\beta_k}; \tag{2.11a}$$

$$V(h^k_{ij}) = \frac{\partial(h^k_{ij})}{\partial R_i} V_{R_i} + \frac{\partial(h^k_{ij})}{\partial R_j} V_{R_j} + \frac{\partial(h^k_{ij})}{\partial R_k} V_{R_k} + \frac{\partial(h^k_{ij})}{\partial \beta_i} V_{\beta_i} + \frac{\partial(h^k_{ij})}{\partial \beta_j} V_{\beta_j} + \frac{\partial(h^k_{ij})}{\partial \beta_k} V_{\beta_k}. \tag{2.11b}$$

From the other side:

$$(s_{ik})_{geo} = (s_{ik})^0 + \Delta(s_{ik}), \tag{2.12a}$$

$$(h^k_{ij})_{geo} = (h^k_{ij})^0 + \Delta(h^k_{ij}). \tag{2.12b}$$

Where:

$$\begin{aligned}(s_{ik})^0 &= F^g_s(X^0_M, Y^0_M, Z^0_M; X^0_i, Y^0_i, Z^0_i; X^0_k, Y^0_k, Z^0_k) = \\ &= \Phi^g_s(B^0_M, L^0_M, H^0_M, B^0_i, L^0_i, H^0_i; B^0_k, L^0_k, Z^0_k);\end{aligned} \tag{2.13a}$$

$$\begin{aligned}(h^k_{ij})^0 &= F^g_h(X^0_M, Y^0_M, Z^0_M; X_i, Y^0_i, Z^0_i; X^0_j, Y^0_j, Z^0_j; X^0_k, Y^0_k, Z^0_k) = \\ &= \Phi^g_h(B^0_M, L^0_M, H^0_M; B^0_i, L^0_i, H^0_i; B^0_j, L^0_j, H^0_j; B^0_k, L^0_k, H^0_k).\end{aligned} \tag{2.13b}$$

$$\begin{aligned}\Delta(s_{ik}) = {}& \frac{\partial(s_{ik})}{\partial X_M}dX_M + \frac{\partial(s_{ik})}{\partial Y_M}dY_M + \frac{\partial(s_{ik})}{\partial Z_M}dZ_M + \frac{\partial(s_{ik})}{\partial X_i}dX_i + \frac{\partial(s_{ik})}{\partial Y_i}dY_i + \frac{\partial(s_{ik})}{\partial Z_i}dZ_i + \\ & + \frac{\partial(s_{ik})}{\partial X_k}dX_k + \frac{\partial(s_{ik})}{\partial Y_k}dY_k + \frac{\partial(s_{ik})}{\partial Z_k}dZ_k = \frac{\partial(s_{ik})}{\partial B_M}dB_M + \frac{\partial(s_{ik})}{\partial L_M}dL_M + \frac{\partial(s_{ik})}{\partial H_M}dH_M + \\ & + \frac{\partial(s_{ik})}{\partial B_i}dB_i + \frac{\partial(s_{ik})}{\partial L_i}dL_i + \frac{\partial(s_{ik})}{\partial H_i}dH_i + \frac{\partial(s_{ik})}{\partial B_k}dB_k + \frac{\partial(s_{ik})}{\partial L_k}dL_k + \frac{\partial(s_{ik})}{\partial H_k}dH_k;\end{aligned} \tag{2.14a}$$

$$\begin{aligned}\Delta(h^k_{ij}) = {}& \frac{\partial(h^k_{ij})}{\partial X_M}dX_M + \frac{\partial(h^k_{ij})}{\partial Y_M}dY_M + \frac{\partial(h^k_{ij})}{\partial Z_M}dZ_M + \frac{\partial(h^k_{ij})}{\partial X_i}dX_i + \frac{\partial(h^k_{ij})}{\partial Y_i}dY_i + \frac{\partial(h^k_{ij})}{\partial Z_i}dZ_i + \\ & + \frac{\partial(h^k_{ij})}{\partial X_j}dX_j + \frac{\partial(h^k_{ij})}{\partial Y_j}dY_j + \frac{\partial(h^k_{ij})}{\partial Z_j}dZ_j + \frac{\partial(h^k_{ij})}{\partial X_k}dX_k + \frac{\partial(h^k_{ij})}{\partial Y_k}dY_k + \frac{\partial(h^k_{ij})}{\partial Z_k}dZ_k = \\ = {}& \frac{\partial(h^k_{ij})}{\partial B_M}dB_M + \frac{\partial(h^k_{ij})}{\partial L_M}dL_M + \frac{\partial(h^k_{ij})}{\partial H_M}dH_M + \frac{\partial(h^k_{ij})}{\partial B_i}dB_i + \frac{\partial(h^k_{ij})}{\partial L_i}dL_i + \frac{\partial(h^k_{ij})}{\partial H_i}dH_i + \\ & + \frac{\partial(h^k_{ij})}{\partial B_j}aB_j + \frac{\partial(h^k_{ij})}{\partial L_j}dL_j + \frac{\partial(h^k_{ij})}{\partial H_j}dH_j + \frac{\partial(h^k_{ij})}{\partial B_k}dB_k + \frac{\partial(h^k_{ij})}{\partial L_k}dL_k + \frac{\partial(h^k_{ij})}{\partial H_k}dH_k.\end{aligned} \tag{2.14b}$$

Partial derivatives appearing in formulae (2.11) and (2.14) are obtained on the basis of formulae (2.6a) and (2.6b).

Using the dependencies

$$(s_{ik})_{lok} = (s_{ik})_{geo}, \tag{2.15a}$$

$$(h^k_{ij})_{lok} = (h^k_{ij})_{geo}, \tag{2.15b}$$

or (2.9) and (2.12) we obtain

$$V(s_{ik}) = \Delta(s_{ik}) + (s^0_{ik})^0 - (s_{ik})', \tag{2.16a}$$

$$V(h^k_{ij}) = \Delta(h^k_{ij}) + (h^k_{ij})^0 - (h^k_{ij})'. \tag{2.16b}$$

Expressions (2.16a) and (2.16b) are really equations for correcting quantities (s) and (h). In order to solve these equations, as already noted, it is necessary to use the principle of adjustment for dependent quantities by the indirect (parametric) method (6).

There is, however, another possible approach. Using formulae (2.11a) and (2.11b) the quantities $V(s_{ik})$ and $V(h^k_{ij})$ appearing in equations (2.16a) and (2.16b) can be replaced by corrections $V_{(R)}$ and $V_{(\beta)}$ to directly measured quantities.

Adjusting a spatial net in this case leads to another standard problem of the method of smallest squares -- of conditional adjustment with unknowns.

Depending on the means of presentation (S_{ik}) and (h^k_{ij}) in adjusting, appear as cartesian unknown geocentric coordinates (X, Y, Z), or geodetic coordinates (B, L, H) of net points.

Installing a spatial net of high accuracy -- in order to eliminate distortion of the net produced by errors in initial data -- we accept as initial data the geodetic coordinates (B, L, H) of two points and the altitude H of the third point.

Independent of what was presented above, a practical solution of the problem encounters difficulties of a practical kind, connected with the designation of the coordinates (B, L, H) of two initial points, and doing so in such a manner that they would correspond accurately to the designated length between these two points. This is a problem which

deserves especially close attention. Here we shall limit ourselves only to a discussion of one of the substantive cases -- from a practical point of view.

Let there be, for one of the net points, which we shall call the beginning, geodetic coordinates $(B, L, H)_0$. Let us assume that we possess the geodetic azimuth A_{01}, the zenith distance β_{01}, and the incline distance D_{01} to adjacent point 1. The question arises, how can the simple geodetic problem be solved in space -- designating the geodetic coordinates $(X, Y, Z)_1$?

Here we shall utilize the formula:

$$\operatorname{tg} A_{01} = \frac{\cos L_0(Y_1 - Y_0) - \sin L_0(X_1 - X_0)}{\cos B_0(Z_1 - Z_0) - \sin B_0[\cos L_0(X_1 - X_0) + \sin L_0(Y_1 - Y_0)]};$$

$$\cos\beta_{01} = \frac{\cos B_0[\cos L_0(X_1 - X_0) + \sin L_0(Y_1 - Y_0)] + \sin B_0(Z_1 - Z_0)}{\sqrt{(X_1 - X_0)^2 + (Y_1 - Y_0)^2 + (Z_1 - Z_0)^2}};$$

$$D_{01}^2 = (X_1 - X_0)^2 + (Y_1 - Y_0)^2 + (Z_1 - Z_0)^2 \qquad (2.17)$$

In the above formulae we can regard as known quantities $(X, Y, Z)_0$, $(B, L, H)_0$, A_{01}, β_{01}, D_{01}, and as unknowns -- (X, Y, Z).

In other words, we have three nonlinear equations with three unknowns. Solving them, for example by the Newton method, there is obtained the required exactitude $(X, Y, Z)_1$. Next with the aid of formula (2.7) the transformation $(X, Y, Z)_1 \rightarrow (B, L, H)_1$ is performed, thanks to which the problem having as a goal the determination of coordinates (B, L, H) for two points, in order to determine sufficiently accurately the net scale, is solved from both the practical and theoretical points of view.

By including also the ellipsoidal height (H) of the third point, we obtain the parameters essential for the entire net.

In the case of our having other astronomic dimensions for the net, or length dimensions, we can consider other possibilities for accurate designation of these seven parameters. These questions, however, will not be considered here.

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